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Nümerik Analize Giriş Final 24.06.2022

1) $f(x)=2\sinh x$, $x_0=-\ln 2$, $x_1=\ln 2$, $x_2=\ln 8$ olduğuna göre $f(0)$ değerini uygun interpolasyon polinomu yardımıyla hesaplayınız

2) $P(x)=\dfrac{1}{\sqrt{1-(x-1)^2}}$ ağırlık fonksiyonu olmak üzere $n=1$ için $\displaystyle\int_0^2 \frac{dx}{\sqrt{1-(x-1)^4}}$ integralini Gauss yöntemiyle hesaplayınız

3) $\displaystyle\int_{-1}^{1} \ln\left(1+\cos\frac{3x\pi}{8}\right)dx$ integralini $n=3$ için dikdörtgen yöntemiyle hesaplayınız

4) $f[x_i, x_{i+1}, \dots, x_{i+m}] = \dfrac{\Delta^m f_i}{h^m\, m!}$ olduğunu gösteriniz. $x_i = x_0 + ih$ dir.

5) x_i, $i=0,1,2$ ayrık noktaları ve $f(x_i)=3i^2-2$ veriliyor. $P_{01}(x)=-1/2+3/2x$, $P_{12}(x)=-\frac{7}{2}+\frac{9}{2}x$ olduğuna göre $P_{012}(0)=?$

6) $\displaystyle\int_{-2}^{2} \sin\left(\frac{x^2\pi}{2}\right)dx$ integralini $n=4$ için Simpson yöntemiyle hesaplayınız.

Not: Sadece dört soru seçerek cevaplandırınız

Başarılar. - N.A.

Final Çözümleri 24.06.2022

1) $f(x)=2\sinh x$, $x_0=-\ln 2$, $x_1=\ln 2$, $x_2=\ln 8$

$f(x)=2\sinh x = e^x-e^{-x}$ olur. $f(x_0)=f(-\ln 2)=-\frac{3}{2}$, $f(x_1)=f(\ln 2)=\frac{3}{2}$ $f(x_2)=f(\ln 8)=\frac{63}{8}$ dir. Ayrık noktalar eşit aralıklı old.dan yani $h=2\ln 2$ old.dan ve $x=0$ $x_0=-\ln 2<0<x_1=\ln 2$ old.dan da $f(0)$ hesaplanmasında ileri fark interpolasyon polinomu yani

$$P_2(x)=P_2(x_0+th)=f(x_0)+t\Delta f(x_0)+\frac{t(t-1)}{2}\Delta^2 f(x_0)$$ polinomu

kullanılması uygundur.

$x=x_0+th$ $x=0$, $h=2\ln 2$ old.dan $0=-\ln 2+t2\ln 2$

$t=\frac{1}{2}$ olur. $\Delta f(x_0)=f(x_1)-f(x_0) \Rightarrow \Delta f(x_0)=\frac{3}{2}+\frac{3}{2}=3$

$\Delta^2 f(x_0)=\Delta f(x_1)-\Delta f(x_0)=f(x_2)-2f(x_1)+f(x_0) \Rightarrow$

$\Delta^2 f(x_0)=\frac{63}{8}-2.\frac{3}{2}-\frac{3}{2}=\frac{27}{8}$ olur. O halde

$\cancel{\left(P_2(0)=P_2\left(-\ln 2+\frac{1}{2}2\ln 2\right)= \quad +\frac{1}{2}.3+\frac{1/2(1/2-1)}{2}.\frac{27}{8}\right)}$

$$P_2(x)=P_2(x_0+th)=-\frac{3}{2}+t.3+\frac{t(t-1)}{2}\frac{27}{8}$$ olur.

$$f(0)\approx P_2(0)=P_2\left(-\ln 2+\frac{1}{2}2\ln 2\right)=-\cancel{\frac{3}{2}}+\cancel{\frac{1}{2}.3}+\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}-1\right)\right)\frac{27}{8}$$
$$=-\frac{27}{64}$$ olur.

$x=0$, $-\ln 2=x_0<0<x_1=\ln 2$ old.da $f(x_0)<f(0)<f(x_1)$ olmalıdır. Gerçekten $-3/2<-\frac{27}{64}<\frac{3}{2}$ old.dan

$\left(-\frac{3.32}{2.32}=-\frac{96}{64} \text{ old.da } -\frac{96}{64}<-\frac{27}{64}<\frac{96}{64}\right)$ ~~old.~~

sonuç doğrudur.

2) $P(x)=\dfrac{1}{\sqrt{1-(x-1)^2}}$ ağırlık fonk. old.dan

$$\int_0^2 \frac{1}{\sqrt{1-(x-1)^2}}\,dx=\int_0^2 \frac{1}{\sqrt{1-(x-1)^2}}\,\frac{1}{\sqrt{1+(x-1)^2}}\,dx=\int_0^2 P(x)\,f(x)\,dx$$

olur. $a=0$, $b=2$, $x=\frac{b+a}{2}+\frac{b-a}{2}t \Rightarrow x=1+t \Rightarrow dx=dt$

$x=0$, $t=-1$, $x=2$ için $t=1$ olur. O halde

$$\int_0^2 P(x)f(x)\,dx=\int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}\,\frac{1}{\sqrt{1+t^2}}\,dt=\int_{-1}^{1}p(t)\,g(t)\,dt \quad \text{olur.}$$

~~n=1 için~~ $\cancel{\int_{-1}^{1} p(t)\,g(t)\,dt \approx c_1 g(t_1) =}$

$\int_{-1}^{1} p(t)\,g(t)\,dt \approx \sum_{k=1}^{n} c_k\,g(t_k)$ olur. $n=1$ old.dan

$\int_{-1}^{1} p(t)\,g(t)\,dt \approx c_1\,g(t_1)$ olur.

$$\left.\begin{aligned} c_1 &= \int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}\,dt \\ c_1 t_1 &= \int_{-1}^{1}\frac{t}{\sqrt{1-t^2}}\,dt \end{aligned}\right\}$$

denklem sisteminden

$$c_1=\int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}\,dt=\arcsin t\Big|_{-1}^{1}=\arcsin 1-\arcsin(-1)=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\pi$$

$$c_1 t_1=\int_{-1}^{1}\frac{t}{\sqrt{1-t^2}}\,dt \Rightarrow \pi\cdot t_1=-\int_{-1}^{1}\frac{-2t}{2\sqrt{1-t^2}}\,dt=-\sqrt{1-t^2}\Big|_{-1}^{1}=0$$

$\pi t_1=0 \Rightarrow t_1=0$ olur. O halde

$$\int_{-1}^{1} p(t)\,g(t)\,dt=\int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}\cdot\frac{1}{\sqrt{1+t^2}}\,dt \approx \pi\cdot\frac{1}{\sqrt{1+0}}=\pi \quad \text{olur.}$$

$$\left(\int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}\,g(t)\,dt=\frac{\pi}{n}\sum_{k=1}^{n} g(t_k)\,,\quad t_k=\cos\frac{(2k-1)\pi}{2}\,,\ k=1,2,\dots,n\right.$$

$$t_1=\cos\frac{(2-1)\pi}{2}=0$$

$$\left. n=1 \text{ için } \int_{-1}^{1}\frac{1}{\sqrt{1-t^2}}\,g(t) \approx \frac{\pi}{1}\cdot g(0)\right)$$

3) $\int_{-1}^{1} \ln\left(1+\cos\frac{3x\pi}{8}\right)dx$, $n=3$ $\quad h=\frac{b-a}{n} \Rightarrow h=\frac{1-(-1)}{3}=\frac{2}{3}$

$x_0=-1$, $x_1=-\frac{1}{3}$, $x_2=\frac{1}{3}$, $x_3=1$, $x_{1/2}=-\frac{2}{3}$, $x_{3/2}=0$, $x_{5/2}=\frac{2}{3}$

$\int_{-1}^{1} \underbrace{\ln\left(1+\cos\frac{3x\pi}{8}\right)}_{f(x)}dx \approx h\sum_{i=1}^{n} f(x_{i/2})$ dır

$f(x_{1/2})=f(-\frac{2}{3})=\ln(1+\frac{\sqrt{2}}{2})$, $f(x_{3/2})=f(0)=\ln 2$

$f(x_{5/2})=f(\frac{2}{3})=\ln(1+\frac{\sqrt{2}}{2})$ dır. $h=\frac{2}{3}$ old.d

$\int_{-1}^{1} f(x)dx \approx \frac{2}{3}\sum_{i=1}^{3} f(x_{i/2})=\frac{2}{3}\left(\ln(1+\frac{\sqrt{2}}{2})+\ln 2+\ln(1+\frac{\sqrt{2}}{2})\right)$ olur

4) $f[x_i, x_{i+1}, \dots, x_{i+m}]=\frac{\Delta^m f_i}{h^m m!}$, $x_i=x_0+ih$

$m=1$ için $f[x_i, x_{i+1}]=\frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}=\frac{\Delta^1 f_i}{h^1 1!}$ old. doğrudur

$m\leq k$ için doğru olsun. Yani

$f[x_i, x_{i+1}, \dots, x_{i+k}]=\frac{\Delta^k f_i}{h^k k!}$ olsun

$f[x_i, x_{i+1}, \dots, x_{i+k}, x_{i+k+1}]=\frac{f[x_{i+1}, \dots, x_{i+k+1}]-f[x_i, \dots, x_{i+k}]}{x_{i+k+1}-x_i}$

$=\cancel{\frac{\Delta^k f_{i+1}-\Delta^k f_i}{h^{k+1}}}$

$=\frac{1}{h(k+1)}\left(\frac{\Delta^k f_{i+1}}{h^k k!}-\frac{\Delta^k f_i}{h^k k!}\right)=\frac{\Delta^k(f_{i+1}-f_i)}{h\cdot h^k\cdot k!(k+1)}=\frac{\Delta^k\Delta f_i}{h^{k+1}(k+1)!}$

$=\frac{\Delta^{k+1} f_i}{h^{k+1}(k+1)!}$ olur.

5) $$P_{012}(0) = \frac{\begin{vmatrix} P_{01}(0) & x_0 \\ P_{12}(0) & x_2 \end{vmatrix}}{x_2 - x_0}, \quad P_{01}(0) = -1/2 \quad P_{12}(0) = -7/2$$

$$P_{012}(0) = \frac{-1/2\, x_2 + \frac{7}{2} x_0}{x_2 - x_0}, \quad x_2 = ? \quad x_0 = ? \quad f_0 = -2,\ f_1 = 1 \quad f_2 = 10$$

$$P_{01}(x) = \frac{\begin{vmatrix} f_0 & x_0 - x \\ f_1 & x_1 - x \end{vmatrix}}{x_1 - x_0} \Rightarrow \frac{\begin{vmatrix} -2 & x_0 - x \\ 1 & x_1 - x \end{vmatrix}}{x_1 - x_0} = \frac{-2(x_1 - x) - (x_0 - x)}{x_1 - x_0} =$$

$$= \frac{-2x_1 + 2x - x_0 + x}{x_1 - x_0} = \frac{3}{x_1 - x_0} \cdot x - \frac{2x_1 + x_0}{x_1 - x_0} = -1/2 + \frac{3}{2} x$$

$$\frac{3}{x_1 - x_0} = \frac{3}{2} \Rightarrow \boxed{x_1 - x_0 = 2}$$

$$\frac{2x_1 + x_0}{x_1 - x_0} = \frac{1}{2} \Rightarrow 2x_1 + x_0 = \frac{1}{2} x_1 - \frac{1}{2} x_0 \Rightarrow \left(2 - \frac{1}{2}\right) x_1 + \left(1 + \frac{1}{2}\right) x_0 = 0$$

$$\to 2x_1 = 2 \Rightarrow x_1 = 1, \quad x_0 = -1$$

$$\frac{3}{2} x_1 + \frac{3}{2} x_0 = 0 \quad \boxed{x_1 + x_0 = 0}$$

olur.

$$P_{12}(x) = \frac{\begin{vmatrix} f_1 & x_1 - x \\ f_2 & x_2 - x \end{vmatrix}}{x_2 - x_1} = \frac{\begin{vmatrix} 1 & x_1 - x \\ 10 & x_2 - x \end{vmatrix}}{x_2 - x_1} = \frac{x_2 - x}{x_2 - x_1} - \frac{10(x_1 - x)}{x_2 - x_1}$$

$$= \frac{x_2 - x - 10x_1 + 10x}{x_2 - x_1} = \frac{9}{x_2 - x_1} x + \frac{x_2 - 10x_1}{x_2 - x_1} = -\frac{7}{2} + \frac{9}{2} x$$

$x_2 - x_1 = 2$, $x_1 = 1$ $\quad x_2 = 2 + 1 = 3$ olur.

$$P_{012}(0) = \frac{-1/2\, 3 + \frac{7}{2}(-1)}{3 - (-1)} = \frac{-3/2 - 7/2}{4} = \frac{-5}{4} \text{ olur.}$$

6) $\int_{-2}^{2} \sin\left(\frac{x^2\pi}{2}\right)dx$ $\quad n=4$ simpson $(4=2m,\ m=2)$

$a=-2,\ b=2 \quad h=\frac{b-a}{n} \Rightarrow h=\frac{2+2}{4}=1$

$x_0=-2,\ x_1=-1,\ x_2=0,\ x_3=1 \quad x_4=2$

$$\int_a^b f(x)dx \approx \frac{h}{3}\sum_{i=1}^{2}\left(f(x_{2i-2})+4f(x_{2i-1})+f(x_{2i})\right)$$

$$\approx \frac{h}{3}\left(f(x_0)+4f(x_1)+f(x_2)+f(x_2)+4f(x_3)+f(x_4)\right)$$

$$\approx \frac{h}{3}\left(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)\right)$$

$f(x_0)=f(-2)=\sin\left(\frac{4\pi}{2}\right)=\sin 2\pi=0$

$f(x_1)=f(-1)=\sin\left(\frac{\pi}{2}\right)=1$

$f(x_2)=f(0)=\sin 0=0$

$f(x_3)=f(1)=\sin\frac{\pi}{2}=1$

$f(x_4)=f(2)=\sin\left(\frac{4\pi}{2}\right)=0\ ,\ h=1$

$\int_{-2}^{2}\sin\left(\frac{x^2\pi}{2}\right)dx=\frac{1}{3}\left(0+4\cdot1+2\cdot0+4\cdot1+0\right)=\frac{8}{3}$ olur